INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 4.12]

POISSON DISTRIBUTION

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A. Paper 1 questions (SHORT)

1. [Maximum mark: 12]

The random variable X has a Poisson distribution with mean 2.5.

(a) Calculate the following probabilities.

$\mathrm{P}(X = 3)$		$\mathrm{P}(3 \le X \le 5)$	
$\mathrm{P}(X \le 3)$		$\mathrm{P}(X \ge 3)$	
$\mathrm{P}(X < 3)$		$\mathrm{P}(X > 3)$	

[6]

(b) Calculate the following conditional probabilities

$\mathrm{P}(X = 3 \mid X \ge 3)$	
$\mathrm{P}(X \le 5 \mid X \ge 3)$	
$\mathrm{P}(X \ge 5 \mid X \ge 3)$	
$\mathrm{P}(X \le 3 \mid X \le 5)$	

[4]

(c) Find the **mode** of X. [2]

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2. [Maximum mark: 10]

The average number of phone calls in a phone centre is 2 phone calls **per minute**.

Find

(a) The probability of 1 phone call in **one** minute. [2]

(b) The probability of 2 phone calls in **one** minute. [2]

(c) The probability of 1 phone call in **two** minutes. [2]

(d) The probability of 2 phone calls in **two** minutes. [2]

(e) The probability of no phone calls in **half a** minute. [2]

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3. [Maximum mark: 8]

The average number of phone calls in a phone centre is 1 phone call per **two** minutes.

Find

(a) The probability of 1 phone call in **two** minutes. [2]

(b) The probability of 1 phone call in **one** minute. [2]

(c) The probability of no phone calls in **one** minute. [2]

(d) The probability of 3 phone calls in **four** minutes. [2]

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4. [Maximum mark: 5]

The average number of phone calls in a phone centre is 2 phone calls **per minute**.

Find

(a) The probability of **exactly** 3 phone calls in one minute. [1]

(b) The probability of **at most** 3 phone calls in one minute. [2]

(c) The probability of **at least** 3 phone calls in one minute. [2]

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5. [Maximum mark: 6]

The average number of phone calls in a phone centre is 2 phone calls **per minute**.

John bets on the number of phone calls in the following minute.

For no phone calls he earns 10 €.

For 1 or 2 phone calls he loses 1 €.

For 3 or more phone calls he loses 2 €,

(a) Find the expected profit per bet for John. [5]

(b) Find the expected profit for John if he bets for 10 times. [1]

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6. [Maximum mark: 6]

The average number of phone calls in a phone centre is 0.5 phone calls **per minute**.

A period of time is called "quiet" if there is no phone call during that period. Find

(a) The probability that the next minute will be "quiet". [1]

(b) The probability that the next 5 minutes will be "quiet" [2]

(c) The probability that in the next 5 one-minute periods only 2 of them will be "quiet". [3]

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7. [Maximum mark: 6]

The random variable X has a Poisson distribution with mean 4. Calculate

(a) $\mathrm{P}(3 \leq X \leq 5)$; [2]

(b) $\mathrm{P}(X \geq 3)$; [3]

(c) $\mathrm{P}(3 \leq X \leq 5 | X \geq 3)$. [2]

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8. [Maximum mark: 4]

*Give your answers to **four** significant figures.*

A machine produces cloth with some minor faults. The number of faults per metre is a random variable following a Poisson distribution with a mean 3. Calculate the probability that a metre of the cloth contains five or more faults.

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9. [Maximum mark: 5]

A supplier of copper wire looks for flaws before despatching it to customers.
It is known that the number of flaws follow a Poisson probability distribution with a mean of 2.3 flaws per metre.

(a) Determine the probability that there are exactly 2 flaws in 1 metre of the wire. [2]

(b) Determine the probability that there is at least one flaw in 2 metres of the wire. [3]

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10. [Maximum mark: 6]

The number of car accidents occurring per day on a highway follows a Poisson distribution with mean 1.5.

(a) Find the probability that more than two accidents will occur on a given Monday. [2]

(b) Given that at least one accident occurs on another day, find the probability that more than two accidents occur on that day. [4]

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11. [Maximum mark: 8]

The number of bus accidents that occur in a given period of time has a Poisson distribution with a mean of 0.6 accidents per day.

(a) Find the probability that at least two accidents occur on a randomly chosen day. [2]

(b) Find the most likely number of accidents occurring on a randomly chosen day. Justify your answer. [3]

(c) Find the probability that no accidents occur during a randomly chosen seven-day week. [3]

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12. [Maximum mark: 6]

(a) Ahmed is typing Section A of a mathematics examination paper. The number of mistakes that he makes, X, can be modelled by a Poisson distribution with mean 3.2. Find the probability that Ahmed makes exactly four mistakes. [2]

(b) His colleague, Levi, is typing Section B of this paper. The number of mistakes that he makes, Y, can be modelled by a Poisson distribution with mean 1.90. Given that X and Y are independent, find the probability that Ahmed makes exactly four mistakes and Levi makes exactly three mistakes. [4]

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13. [Maximum mark: 7]

Let $X \sim \text{Po}(3)$ and $Y \sim \text{Po}(1)$. Then $X + Y \sim \text{Po}(4)$

(a) Find the probability $P(X + Y = 3)$ [1]

(b) Complete the following table (give your answers in 5dp)

a	b	$P(X = a)$	$P(Y = b)$	$P(X = a \text{ and } Y = b)$
0	3	0.04979	0.06131	0.00305
1	2	0.14936	0.18394	
2	1			
3	0		0.36788	0.08242

[4]

(c) Find the sum of the results in the last column above. State a comment. [2]

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14. [Maximum mark: 6]

Let $X \sim \text{Po}(3)$ and $Y \sim \text{Po}(1)$.

(a) Find the probability that the sum $X + Y$ is equal to 3. [1]

(b) Given that $X + Y = 3$, find the probability that $X = 1$ and $Y = 2$. [2]

(c) Given that $X + Y = 3$, find the probability that $X < Y$. [3]

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15. [Maximum mark: 6]

Let $A \sim \text{Po}(3)$ and $B \sim \text{Po}(4)$.

(a) Find the probability that the sum $A + B$ is equal to 5. [2]

(b) Find the probability that the sum $A + B$ is less than 5. [2]

(c) Find the probability that the sum $A + B$ is more than 5. [2]

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16. [Maximum mark: 8]

The number of accidents in town A follows Poisson with mean 4 accidents per day.

The number of accidents in town B follows Poisson with mean 5 accidents per day.

(a) Find the probability that **exactly** 4 accidents occur in town A and exactly 5 accidents in town B during one day. [3]

(b) Find the probability that a **total** number of 10 accidents occur in both towns during one day. [2]

(c) Find the probability that a **total** number at most 50 accidents occur in both towns during a seven-day week. [3]

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17. [Maximum mark: 12]

*Give all numerical answers to this question correct to **three** significant figures.*

Two typists were given a series of tests to complete. On average, Mr Brown made 2.7 mistakes per test while Mr Smith made 2.5 mistakes per test. Assume that the number of mistakes made by any typist follows a Poisson distribution.

(a) Calculate the probability that, in a particular test,

(i) Mr Brown made **two** mistakes;

(ii) Mr Smith made **three** mistakes;

(iii) Mr Brown made **two** mistakes and Mr Smith made **three** mistakes. [6]

(b) Calculate the probability that Mr Brown and Mr Smith made a combined total of **five** mistakes. [2]

(c) In some test, Mr Brown and Mr Smith made a combined total of **five** mistakes. Calculate the probability that Mr Brown **made fewer mistakes** than Mr Smith. [4]

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18. [Maximum mark: 8]

(a) Patients arrive at random at an emergency room in a hospital at the rate of 15 per hour throughout the day. Find the probability that 6 patients will arrive at the emergency room between 08:00 and 08:15. [3]

The emergency room switchboard has two operators. One operator answers calls for doctors and the other deals with enquiries about patients. The first operator fails to answer 1% of her calls and the second operator fails to answer 3% of his calls. On a typical day, the first and second telephone operators receive 20 and 40 calls respectively during an afternoon session.

(b) For each operator, find the average number of calls they fail to answer. [2]

(c) Using the Poisson distribution find the probability that, between them, the two operators fail to answer two or more calls during an afternoon session. [3]

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B. Paper 2 questions (LONG)

Combination of POISSON with BINOMIAL and NORMAL distributions

19. [Maximum mark: 10]

On a particular road, serious accidents occur at an average rate of two per week and can be modelled using a Poisson distribution.

(a) What is the probability of exactly eight serious accidents occurring during a particular four-week period? [2]

(b) What is the probability of more than eight serious accidents occurring during a particular four-week period? [2]

(c) What is the probability of at least eight serious accidents occurring during a particular four-week period? [2]

(d) Assume that a year consists of thirteen periods of four weeks. Find the probability that in a particular year, there are more than nine four-week periods in which at least eight serious accidents occur. [4]

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20. [Maximum mark: 13]

The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day.

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days). [3]

(b) Determine the probability that there are more than three breakdowns during the month of June. [2]

(c) Determine the probability of no breakdowns during the first five days of June. [2]

(d) Find the probability that the first breakdown in June occurs on June 3rd. [3]

(e) It costs 1850 euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June. [1]

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June. [2]

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21. [Maximum mark: 12]

The distance travelled by students to attend Gauss College is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km.

(a) (i) Find the probability that the distance travelled to Gauss College by a randomly selected student is between 4.8 km and 7.5 km.

(ii) 15% of students travel less than d km to attend Gauss College. Find the value of d. [5]

The number of telephone calls, T, received by Euler College each minute can be modelled by a Poisson distribution with a mean of 3.5.

(b) (i) Find the probability that at least three telephone calls are received by Euler College in **each** of two successive one-minute intervals.

(ii) Find the probability that Euler College receives 15 telephone calls during a randomly selected five-minute interval. [7]

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22. [Maximum mark: 15]

The time, T minutes, spent each day by students in Amy's school sending text messages may be modelled by a normal distribution with a mean of 12.9 minutes and a standard deviation of 5.50 minutes.

(a) Find the percentage of the students that spend less than 10 minutes per day. Give your answer to the nearest whole number. [4]

The number of text messages received by Amy during a fixed time interval may be modelled by a Poisson distribution with a mean of 6 messages per hour.

(b) Find the probability that Amy will receive exactly 8 messages between 16:00 and 18:00 on a random day. [3]

(c) Given than Amy has received at least 10 messages between 16:00 and 18:00 on a random day, find the probability that she received 13 messages during that time. [4]

(d) During a 5-day week, find the probability that there are exactly 3 days when Amy receives no messages between 17:45 and 18:00. [4]

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